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Determination of Young's Modulus from Depth Sensing Vickers Indentation Tests

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Abstract

A new semiempirical formula is proposed for the determination of Young's modulus of material from Depth Sensing Vickers Indentation tests. The parameters used in this equation can be determined from the indentation curves with high accuracy. This formula is consistent with that calculated on the basis of the model of the elastic stress field developed in the material around the Vickers indenter.

1. Introduction

Depth Sensing Indentation (DSI) testing is widely used for the determination of mechanical properties of materials. The indentation method is preferred because relatively small amounts of testing material are needed and there are no strict requirements for the shape of the samples, moreover the measurements can be performed without the destruction of the samples. The DSI test consists of two periods. During the loading period of the test the indenter penetrates into the surface of the sample at constant velocity till the load reaches its maximum value (P_m) and the same velocity is applied in the unloading period when the pyramid moves upwards. During the test the load (P) is registered as a function of the penetration depth (h). Fig.1 shows a schematic load-depth curve. During unloading the elastic relaxation of the material investigated is occurring therefore the elastic properties of the material can be determined from the unloading curve. It's not easy to obtain the elastic parameters from this curve because the shape of the unloading curve depends on both the elastic and the plastic properties of the material. This can be explained in the following way. During the loading period a plastic zone is developing under the indenter therefore the elastic stresses and deformations relaxed during unloading depend on both the plastic and elastic properties of the material. Methods [1-5] found in the literature for the calculation of Young's modulus from indentation tests are based on the theoretical solution of the Boussinesq problem given by Sneddon [6] who determined the load—indentation depth functions for various indenters in an elastic half space. The most frequently used one among these methods was developed by Oliver *et al.* [5]. They elaborated an iterative procedure for the determination of the reduced Young's modulus, E_r, ($E_r = \frac{E}{1-\nu^2}$, where E is the Young's modulus and ν is the Poisson's ratio). The uncertainty of parameters used in the calculation is relatively high and the procedure is complicated. In the present paper a new method is given for the determination of Young's modulus. A technical advantage of the new method is that the parameters used in the formula of Young's modulus can be determined at high accuracy from the indentation curves.

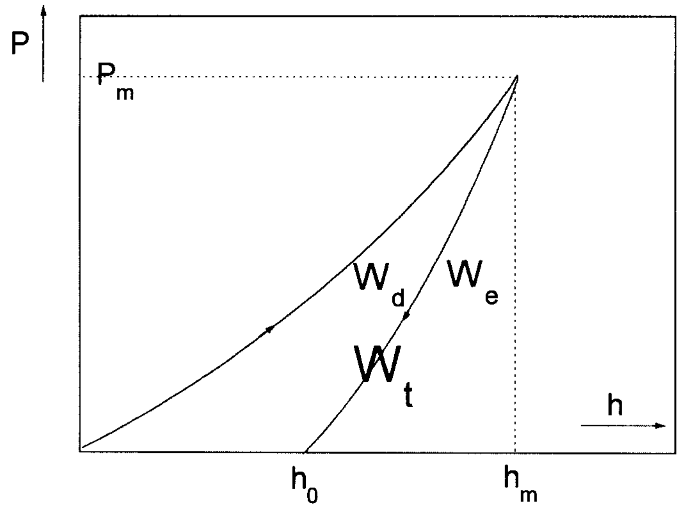

Fig. 1: The schematic load-depth curve.

2. Experimentals:

In our indentation measurements the indenter was a standard Vickers pyramid. The measurements were carried out in the macrohardness region (P_m=50-200N). The indentation velocity was about 1-10μm/s. Measurements were performed on various materials: metals (steel, 99.999% pure Al, Cu, Mg, Ni), soda lime silica glass, sodium chloride, polypropylene, apatite-mullite glass-ceramic, silicon, titanium-oxide ceramic, tetragonal zirconia polycrystalline ceramic sample containing 10 mol % CeO_2 (Ce-TZP) and Si_3N_4 ceramics with three different composition ($x_1$wt%Si_3N_4 $x_2$wt%AlN $x_3$wt%Al_2O_3 $x_4$wt%Y_2O_3; for material A: x_1=90, x_2=0, x_3=4, x_4=6; for material B: x_1=87, x_2=4, x_3=4, x_4=5; for material C: x_1=90.9, x_2=0, x_3=3, x_4=6.1) sintered to different densities.

3. Results and discussion:

3.1. The connection between the parameters of the indentation curves and the Young's modulus

For all the materials investigated in the loading period the load-depth function can be described by a quadratic polinom:

$$P = c_2 h + c_3 h^2 . \tag{1}$$

For the unloading period the load-depth function also satisfies a quadratic equation:

$$P = c_2^*(h - h_0) + c_3^*(h - h_0)^2 , \tag{2}$$

where P is the load, h is the penetration depth and h_0 is the residual indentation depth after removing the punch, c_2, c_3, c_2^*, c_3^* are fitting parameters. The response of material to indentation can be characterised energetically. The total indentation work equals to the area under the loading curve (W_t) (see Fig.1). During unloading the elastic portion of this work can be regained, it equals to the area under the unloading curve (W_e). The difference between the two works gives the energy dissipated during the indentation cycle (W_d). $c_3, c_3^*, W_e/W_t$ are the most important load-independent parameters which can be determined from the indentation curves with high accuracy. According to the experimental results the following relationship is valid for all the materials investigated:

$$\frac{W_e}{W_t} = \sqrt{\frac{c_3}{c_3^*}} . \tag{3}$$

The purpose of this paper is to express the Young's modulus with the indentation parameters. According to the theoretical investigation of Sneddon [6] the load—indentation depth function for an ideally elastic material can be described in both the loading and the unloading period by the same equation

$$P = c_3 h^2 = c_3^* h^2 , \tag{4}$$

with

$$c_3 = c_3^* = \frac{E}{2(1-\nu^2)} \cdot \frac{\alpha_0}{\gamma^2} \cdot \tan \Psi , \tag{5}$$

where E is the Young's modulus, ν is the Poisson's ratio, α_0 is a constant depending on the geometry of the indenter (α_0=2 for Vickers indenter), $\gamma = \frac{\pi}{2}$, ψ is the semi-angle of the indenter (74.05° for Vickers indenter) [2, 3]. From Eq.5 the Young's modulus of ideally elastic materials can be given in the form:

$$E = \alpha c_3^* , \tag{6}$$

with

$$\alpha = 0.71(1-\nu^2) . \tag{7}$$

Empirical evidence shows that Eq.6 is not valid for elasto-plastic materials where the deformation is not ideally elastic. In these cases a plastic zone is developing under the indenter and due to this the elastic stresses and deformations are lower than they would be in an ideally elastic material at the same E and P_m. Consequently the elastic relaxation is also smaller than in the ideally elastic case, therefore αc_3^* is higher than E. A relationship is assumed between E and αc_3^* in the form:

$$E = \alpha c_3^* \beta , \tag{8}$$

where β factor must be less than 1 for elasto-plastic materials and it must depend on the elastic-plastic properties of materials. According to the experimental results the ratio of the elastic and total works performed during the indentation cycle can be used as β, i.e. Eq.8 can be written in the following form:

$$E = \alpha c_3^* \frac{W_e}{W_t}. \tag{9}$$

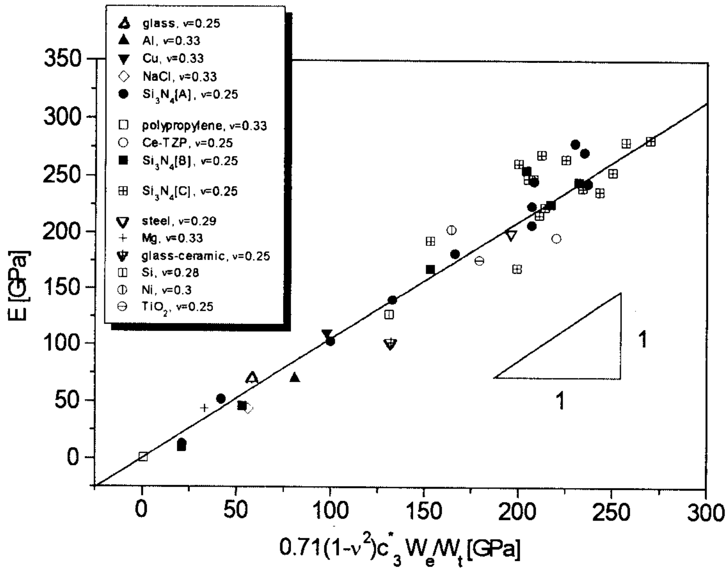

Fig.2.: Young's moduli determined by conventional methods vs. those calculated on the basis of Eq.9.

Eq.9 satisfies the condition of the ideally elastic material because for this material $W_e/W_t=1$. Fig.2 shows that the Young's moduli calculated on the basis of Eq.9 agree relatively well with those measured by conventional mechanical methods (e.g. tensile, compressive and four point bending tests) for a broad variety of materials. For the calculation of α the Poisson's ratios were obtained from the literature. Eq.9 satisfies the condition of the ideally plastic material where $E \to \infty$. It is not obvious from Eq.9 because although $c_3^* \to \infty$, W_e/W_t tends to zero. However, rearranging Eq.9 with the help of Eq.3 we get

$$E = \alpha c_3 \frac{W_t}{W_e}. \tag{10}$$

Eq.10 implies $E \to \infty$ for the ideally plastic limiting case, since c_3 is a finite number and W_t/W_e tends to infinity.

3.2. Determination of Young's modulus from the elastic energy regained during unloading.

In the following the new formula of Young's modulus (Eq.10) is verified by calculating the elastic energy associated with the stress field building up beneath the indenter. The most widely used model of the elastic stress field in materials around Vickers indenter is the Yoffe's model [8]. In spherical polar co-ordinates (r, θ, ϕ) the elastic stress components of this model are given by:

$$\left.\begin{aligned}
\sigma_{rr} &= \frac{P}{2\pi r^2}\left[1-2\nu-2(2-\nu)\cos\theta\right]+\frac{B}{r^3}4\left[(5-\nu)\cos^2\theta-(2-\nu)\right],\\
\sigma_{\theta\theta} &= \frac{P}{2\pi r^2}\frac{(1-2\nu)\cos^2\theta}{(1+\cos\theta)}+\frac{-B}{r^3}2(1-2\nu)\cos^2\theta,\\
\sigma_{\phi\phi} &= \frac{P(1-2\nu)}{2\pi r^2}\left[\cos\theta-\frac{1}{1+\cos\theta}\right]+\frac{B}{r^3}2(1-2\nu)(2-3\cos^2\theta),\\
\tau_{r\theta} &= \frac{P(1-2\nu)}{2\pi r^2}\frac{\sin\theta\cos\theta}{(1+\cos\theta)}+\frac{B}{r^3}4(1+\nu)\sin\theta\cos\theta,\\
\tau_{r\phi} &= \tau_{\theta\phi}=0,
\end{aligned}\right\} \tag{11}$$

where parameter B depends on the mechanical properties of the material investigated and the load applied. In elasto-plastic materials a plastic zone is developing under the punch where Eqs.11 are not valid. This plastic zone is considered as a hemispherical hydrostatic core with a radius equals to the half-diagonal of the contact impression [8-10]. In this hydrostatic core the pressure is taken as the mean contact pressure beneath the pyramid:

$$H=\frac{P_m}{2a^2}, \tag{12}$$

where P_m is the maximum applied load during the indentation cycle and a is the half diagonal of the Vickers pattern.

The normal (σ_{rr}) and shear ($\tau_{r\theta}$) components of the stress field must be continuous at the zone boundary, and B can be determined in Eqs.11 from these conditions. A single value of B can not satisfy these conditions exactly, however the average difference between the stress components calculated for both sides of the interface of the hemispherical zone is minimum if $0.019Ha^3$ is chosen as the value of parameter B [11].

The elastic energy regained during unloading (W_e) can be calculated with the help of the above stress field [11]:

$$W_e=g(\nu)\cdot\frac{H^2a^3}{E}, \tag{13}$$

where $g(\nu)$=2.24, 2.01 or 1.77 as ν=0.25, 0.29 or 0.33.

If H and a are expressed by the parameters of the indentation curves the following relationship is obtained for the Young's modulus [11, 12]:

$$E=0.214g(\nu)\frac{1}{\sqrt{k}-\frac{\varepsilon}{2}\frac{W_e}{W_t}}c_3\frac{W_t}{W_e}, \tag{14}$$

where ε=0.75 and k shows how large the quadratic term in the maximum load ($kP_m=c_3h_m{}^2$). Fig.3. shows that the Young's moduli determined from our previous semi-empirical formula (Eq.10) agree well with those obtained from the Yoffe's elastic stress field (Eq.14) for all the materials investigated.

Consequently the above theoretical considerations verify the applicability of the semiempirical formula of Eq.10 for the determination of the Young's modulus.

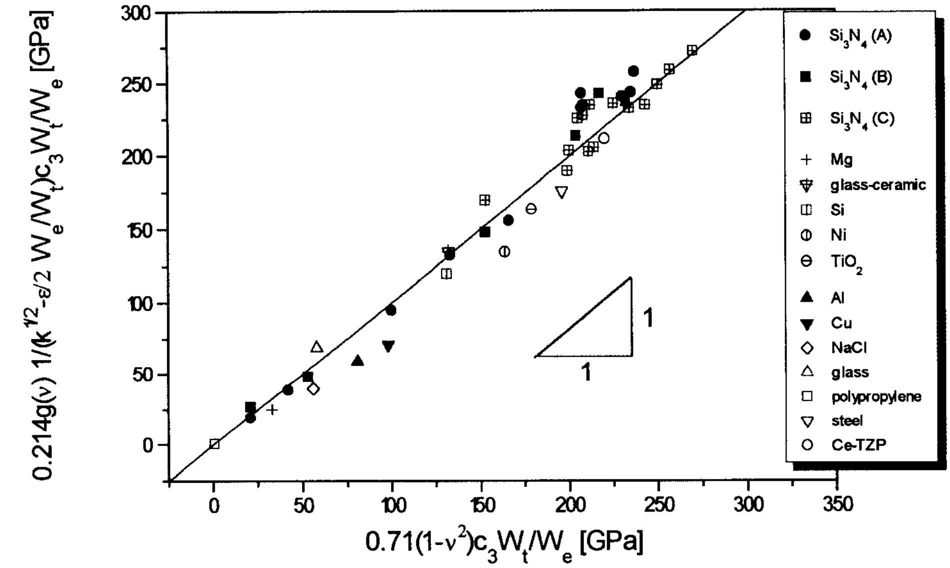

Fig.3: The Young's moduli calculated from Eq.14 vs. the moduli determined on the basis of Eq.10.

4. Conclusions:

A new empirical formula is proposed for the determination of Young's moduli of materials from DSI tests. This formula is basically consistent with that calculated according to the Yoffe's elastic stress field.

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